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Thermoacoustic Oscillations in Resonators

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Thermoacoustic processes involve heat and sound interactions. Their appearances and applications range from combustion instabilities to novel refrigerators and imaging techniques. The focus of this paper is on thermoacoustic oscillations occurring in gases inside chambers. The conversion of heat to sound energy in resonators generally requires in-phase fluctuations of acoustic pressure and heat addition, whereas in specially designed systems the supplied sound can pump heat along solid surfaces from low to high temperature regions. Fundamentals, investigation methods, and some applications of thermoacoustic oscillating phenomena in resonators are reviewed in this paper.

Keywords: *Thermoacoustics; oscillations; wave; Rijke tube*

1 Introduction

Intensive acoustic fluctuations can appear in gases inside chambers (resonators) with heat release under suitable conditions. These phenomena represent important engineering problems in many industrial applications ranging from burners and rocket motors to thermoacoustic engines and refrigerators.

The main cause for thermoacoustic instabilities in resonators is the presence of unsteady heat addition component that fluctuates in phase with acoustic pressure. This requirement is known as Rayleigh criterion [1]. The quantitative form for the heat-sound energy transformation under an assumption of uniform medium in a chamber and small-amplitude oscillations was derived by [2] where ΔE is the amount of acoustic energy generated from heat during an acoustic cycle inside a resonator, p' and p_0 are the acoustic and mean pressure, respectively, $\dot{q}'$ is the heat addition rate per unit volume of the resonator, and γ is the gas specific heat ratio. The integration in Eq. (1) is carried out in time over the cycle period T and in space over the resonator volume V. If thermoacoustically converted power exceeds acoustic losses, then amplitudes of excited acoustic modes will grow in time:

$$\Delta E = \frac{\gamma - 1}{\gamma p_0} \int_V \int_T p' q' \, dt \, dv \qquad (1)$$

There is a variety of system configurations where thermoacoustic energy transformation can occur. Some of them are illustrated in basic forms in Fig. 1. A duct with mean flow and a compact heat source (heater or flame) is known as a Rijke tube (Fig. 1a); it can serve as an imitation of some combustion chambers. A time delay between fluctuating velocity and heat addition rate is essential for exciting acoustic modes in this system. This tube was first described by Rijke (1859) and later was used by many researchers as a convenient setup for studying thermoacoustic instabilities [3].

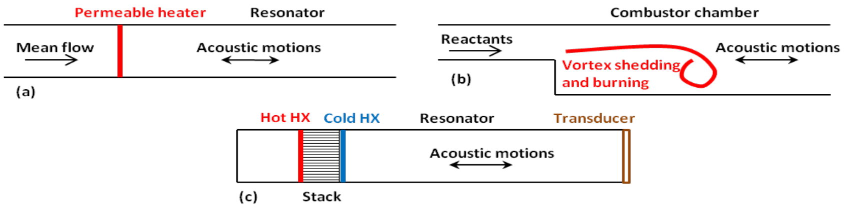

Figure 1. Schematics of systems where thermoacoustic oscillations can occur: (a) Rijke tube, (b) dump combustor, (c) thermoacoustic prime mover.

Another type of thermoacoustic instabilities, involving vortex shedding and combustion, appears in resonators with sudden changes in geometry and reacting flows. These instabilities are related to liquid-fuel motors with premixed combustors, as well as to various industrial burners and gas turbines. One schematic of a process of this sort is shown in Fig. 1b. A vortex generated at the flameholder (a rearward-facing step in Fig. 1b) consists of cold unburnt reactants from an incoming flow and hot products from a recirculation zone [4]. Upon the impingement of such a vortex on a downstream structure (or a wall of the combustion chamber) or after a certain induction time defined by chemical and hydrodynamic properties, a rapid mixing of hot and cold flow components occur, followed by fast heat release due to vortex burning. This sudden heat addition acts as an acoustic disturbance, and resulting acoustic waves influence the vortex shedding process, creating a system with a feedback.

A schematic of a standing-wave thermoacoustic prime mover is shown in Fig. 1c. At sufficiently large temperature gradients inside a porous insert (often called a stack), the resonator acoustic modes may become unstable [5]. The generated acoustic power can be used to produce electricity or pump heat.

The next sections of this paper review several important phenomena and systems involving thermoacoustic oscillations in resonators, including combustion instabilities, Rijke tubes and thermoacoustic engines. It should be noted that besides thermoacoustic processes in devices oriented on energy conversion, there are other emerging technologies related to heat-sound interactions, such as separation of gases in mixtures [5], thermoacoustic hyperthermia and tomography [6], thermoacoustic transducers [7], and sound absorbers [8].

2 Combustion Instabilities

Acoustic-combustion instabilities occurring in combustion chambers are usually quite harmful for the system operation, because they may lead to intensive vibrations, unacceptable for propulsion or power systems, and to enhanced heat transfer, which can overheat the structure. These phenomena were identified a long time ago with introduction of aerospace engines. It still remains an important engineering subject, since in the development stage of rocket motors and modern gas turbines (especially those operating in lean-fuel regimes) practically all of them experience some sort of thermoacoustic instability. On the other hand, there are devices, such as a pulsed combustor [9], where oscillations are essential for normal operation.

The problem of the acoustic instabilities in combustors attracted a lot of attention by many researchers; and extensive theoretical, experimental, and numerical studies were undertaken [10-16]. A common approach to simplified (reduced-order) analysis of such instabilities is to apply an acoustic modal expansion for fluctuating fluid and flow properties and to use models for heat addition that depend on pressure and velocity fluctuations. Some basic elements of such modeling are illustrated in the next section on Rijke tubes. Due to high complexity of actual physical processes in real-world combustors and motors, more or less accurate models must also include effects of mean flow, medium non-uniformity, nonlinearities, noise, etc., which lead to significant complication of the analysis.

More detailed simulations of acoustic-combustion instabilities can be also conducted with the use of modern computational fluid dynamics (CFD) solvers [17], [18]. However, such simulations remain computationally expensive and still require extensive validation due to simplified nature of some models employed by these programs (e.g., for turbulence and chemistry in reacting flows).

Comprehensive experimental studies of combustion instabilities require sophisticated equipment, such as laser-based diagnostic systems [19] and are usually conducted in simplified setups that may not accurately represent real devices (such as motors of large rockets). Nevertheless, these experiments provide useful insight on heat-sound interactions and can be used for validating numerical tools. One of the practically important experimental directions is to measure flame response to forced acoustic oscillations with the purpose to establish relations between acoustic fluctuations and unsteady heat addition (flame transfer functions), which can be subsequently used in simplified modeling of acoustic-combustion instabilities [20].

3 Rijke Tube

Perhaps the most commonly used setup for demonstrating thermoacoustic phenomena and studying fundamentals of thermoacoustic instabilities, both experimentally and theoretically, is the Rijke tube (Fig. 1a). It represents a resonator, usually a straight tube, with a permeable heater. Mean flow through a Rijke tube is the essential feature; it can be caused either by natural convection (for vertically oriented tubes) or by external means (e.g., air blower in horizontal configurations). At some values of the main system parameters (supplied heat, heater position and mean flow rate), self-excited acoustic oscillations appear.

In a simple approximation for a Rijke tube, the one-dimensional wave equation for acoustic pressure p' in the presence of heat addition with rate $\dot{q}'$ per unit volume can be written as follows [21]:

$$\frac{\partial^2 p'}{\partial t^2} - a^2 \frac{\partial^2 p'}{\partial x^2} = (\gamma - 1)\frac{\partial \dot{q}'}{\partial t} \qquad (2)$$

where t is time, a is the speed of sound, and x is the coordinate along the resonator. Acoustic motions in the chamber can be presented using Galerkin technique for the acoustic mode decomposition [22],

$$p'(x,t) = p_0 \sum \eta_n(t)\psi_n(x) \qquad (3)$$

$$u'(x,t) = \sum \frac{1}{\gamma k_n^2}\frac{d\eta_n}{dt}\frac{d\psi_n}{dx} \qquad (4)$$

where $\eta_n(t)$ is the time-varying amplitude of the n$^{\text{th}}$ mode, $\psi_n(x)$ is the pressure mode shape, and k_n is the modal wave number. The mode shapes and corresponding natural frequencies can be determined by substituting acoustic modes from Eq. (3) into Eq. (2) and solving with the zero right-hand side. For example, in the case of uniform medium and constant cross-sectional area and the simple pressure release boundary conditions $p'(0,t) = p'(L,t) = 0$, the waveforms will be $\psi_n = \sin(k_n x)$ with $k_n = \pi n / L$, where L is the tube length.

In the case of a compact heater, the unsteady linearized component of convective heat transfer rate can often presented as follows [23],

$$\frac{\dot{q}'}{\dot{Q}_0} = A\frac{u'(x,t-\tau)}{u_0}\delta(x - x_h) \qquad (5)$$

where $\dot{Q}_0$ and u_0 are the steady components of the heat addition rate and flow velocity, δ is the delta function, x_h is the heater location along the tube, τ is the delay due to thermal inertia, and A is the magnitude of the transfer function between velocity and heat fluctuations. The form of Eq. (5) is similar to the well-known $n-\tau$ model commonly used for simple analysis of thermoacoustic instabilities [24].

Considering small cycle-to-cycle variations and utilizing Rayleigh criterion (Eq. 1), one can determine a dependence of acoustic power converted from heat for the first acoustic mode in a Rijke tube

$$\dot{E}_{conv} \sim \frac{\dot{Q}_0}{u_0} A \sin(2\pi x_h / L)\sin(\omega\tau) \qquad (6)$$

where ω is the angular frequency of oscillations. With the time lag often being small, Eq. (6) implies that positive heat-to-sound energy conversion for the first mode can occur only if the heater is positioned in the upstream half of the tube, and the heater optimal location is at one-quarter tube length from the tube upstream end. Accounting for acoustic losses, non-uniform medium and other effects, reasonably good predictions can be made for the sound onset in an actual Rijke tube. An example of a simplified comparison of theoretical results and test data is shown in Fig. 2c. These data also include results for a segmented Rijke tube consisting of two tubes with different diameters (Fig. 2a); such a system was found to have substantially lower heating requirements for exhibiting thermoacoustic instability.

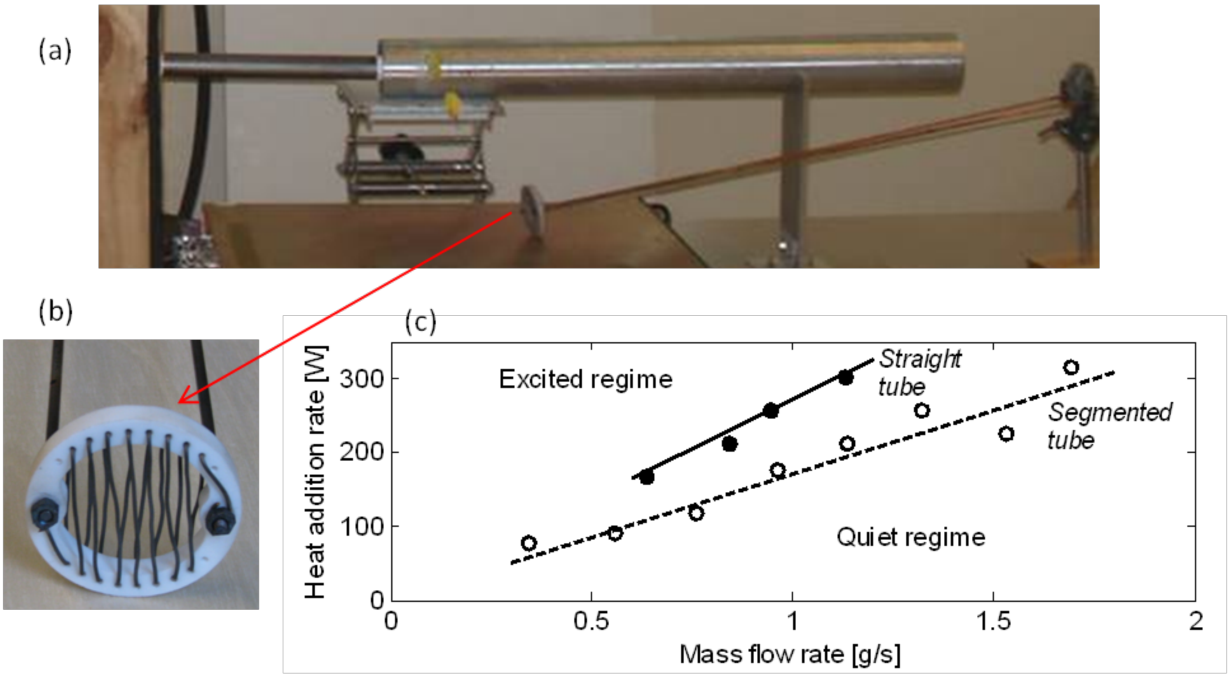

Figure 2. (a) Electrically driven segmented Rijke tube with controllable mean flow. (b) Nichrome wire heater. (c) Boundaries between quiet and excited regimes in constant-diameter Rijke tube (filled circles) and segmented Rijke tube (open circles). Symbols show experimental data points; lines correspond to theoretical predictions.

Besides the onset of sound, a variety of nonlinear thermoacoustic effects are observed in Rijke tube systems, including hysteresis, limit-cycle saturation, noise-induced triggering of instability, interactions between harmonics, and so on. An example of hysteresis in the stability boundary is given in Fig. 3a. Much higher heating power is required to initiate sound in comparison with the transition back to the quiet state. This hysteresis loop resembles a combination of subcritical Hopf and fold bifurcations in a dynamical system (Fig. 3b). Similar phenomena have been also observed in actual combustion systems prone to thermoacoustic instabilities. In recent years, some efforts have been undertaken to apply nonlinear analysis of dynamical systems to model such effects in Rijke tubes with a hope of eventually extending these models to predict and control performance of real-world devices [25-27].

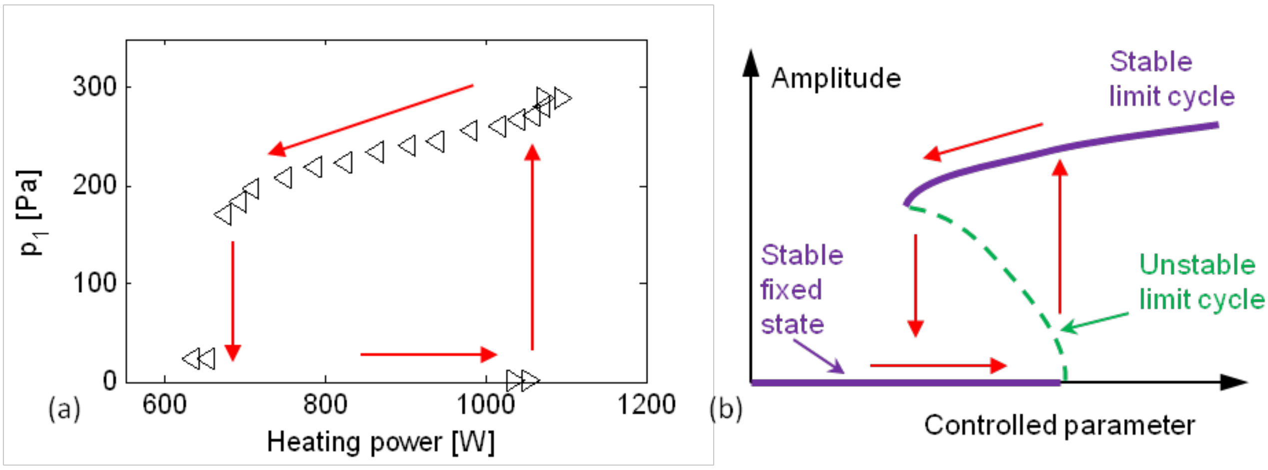

Figure 3. (a) Acoustic pressure amplitude and transitions between quiet and excited states in Rijke tube (Matveev 2003). Variation of heat supply rate: ►, increase; ◄, decrease. (b) Subcritical Hopf and fold bifurcations.

4 Thermoacoustic Engines

Besides generally harmful thermoacoustic instabilities in rockets and burners, thermoacoustic oscillations can be also beneficially applied to convert heat into mechanical and then to electrical power or to pump heat. Novel devices developed for this purpose are known as thermoacoustic engines, which comprise both prime movers and refrigerators. In these systems, thermal interactions between oscillating gas parcels and solid surfaces is essential, and most devices of this sort employ s some sort of porous material (called stack or regenerator) with large surface area. A schematic of a simple standing-wave engine is shown in Fig. 4. The acoustic power is generated in the stack in the presence of externally maintained, sufficiently large temperature gradient. At the proper location of the stack inside the resonator, the heat is transported to gas parcels oscillating in the fundamental acoustic mode at the moment of their compression and extracted at the moment of rarefaction. Therefore, acoustic power is generated, according to Rayleigh criterion (Eq. 1). Besides simple standing-wave engines, more complicated and more efficient traveling-wave engines with closed loops, as well as cascade engines, have been introduced [31-33], some of which experimentally demonstrated high second-law efficiencies approaching 50%. With a source of acoustic power, it is also possible to invert the thermoacoustic process and pump heat from a low-temperature space to a high-temperature sink, thus producing refrigerating effect [34].

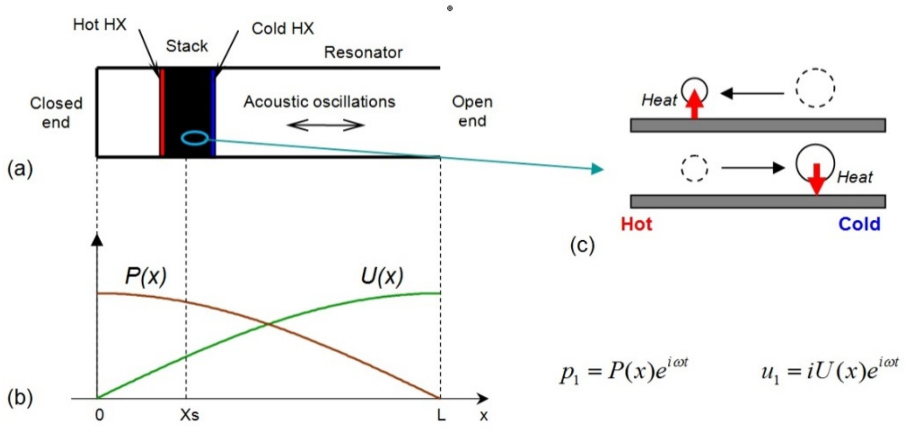

Figure 4. (a) Schematic of a standing-wave thermoacoustic engine. (b) Ideal acoustic pressure and velocity waveforms. (c) Thermal interaction between an oscillating gas parcel and stack plate with strong temperature gradient during one acoustic cycle.

The initial fundamental analysis of thermoviscous interactions between oscillating gas and solid surface was carried out by [28]. The adaptation of this linearized theory for practical calculations, development of a numerical software DELTAE (freely available), and construction and testing or prototype thermoacoustic systems was accomplished at Los Alamos National Laboratory [29], [30]. One of the major theoretical results is the acoustic power $\dot{W}_{ac}$ produced by inviscid ideal gas oscillating near a two-sided solid surface under a mean temperature gradient $\left|\nabla T\right|$,

$$\dot{W}_{ac} = \frac{1}{2}\delta_k BL\omega \frac{\gamma-1}{\gamma}\frac{p_1{}^2}{p_0}\left(\frac{\left|\nabla T\right|}{\left|\nabla T_{cr,id}\right|}-1\right) \qquad (7)$$

where B and L are the width and length of the plate, and p_1 is the acoustic pressure amplitude. The ideal critical temperature gradient $\left|\nabla T_{cr,id}\right|$ and thermal penetration depth δ_k in Eq. (8) are defined as follows,

$$\left|\nabla T_{cr,id}\right| = \frac{\omega\, p_1}{\rho_m c_p u_1} \qquad (8)$$

$$\delta_k = \sqrt{\frac{2K}{\omega}} \qquad (9)$$

where ρ_0 is the mean density, c_p is the specific heat of the gas at constant pressure, u_1 is the acoustic velocity amplitude, and K is the thermal diffusivity. The physical meaning of the thermal penetration depth is the length over which heat can propagate during one acoustic cycle. The critical temperature gradient $\nabla T_{cr,id}$ corresponds to the condition of sound onset in the idealized system. These equations can be used to roughly evaluate the necessary temperature difference and the amount of acoustic power that can be generated. For more accurate estimations, one needs to account for viscous effects in the stack and acoustic losses in other system elements [35].

Although the linearized thermoacoustic theory is found to be in reasonably good agreement with experimental results, most practical thermoacoustic systems operate (or are expected to operate) in high amplitude regimes. Accordingly, new scientific and engineering challenges arise due to novel complex and interesting phenomena, such as nonlinearities, acoustic streaming, interface conditions, turbulence, transient regimes, integration with electroacoustic transducers, and so on. Investigations of some of these effects have started, but more experimental and computational studies are certainly required [36-39].

With regard to practical applications of thermoacoustic engines and refrigerators, thermoacoustic cryocoolers are already available on a commercial basis from several companies, such as Qdrive and Sunpower. Prototype thermoacoustic systems have been tested for a variety of applications, including natural gas liquefaction [40] remotely located power systems and refrigerators [41], devices for harvesting solar energy and waste heat [42], cooking stoves that can produce electricity and cooling, and so on. There are also efforts aimed at developing miniature thermoacoustic engines that can provide power or cooling for remote sensors when a heat source is available, thus eliminating a need for battery replacement [43]. Figure 5 shows a demonstrator of a small-scale engine (similar to that in Fig 1c but without an electroacoustic transducer), as well as acoustic pressure amplitude measured inside the resonator.

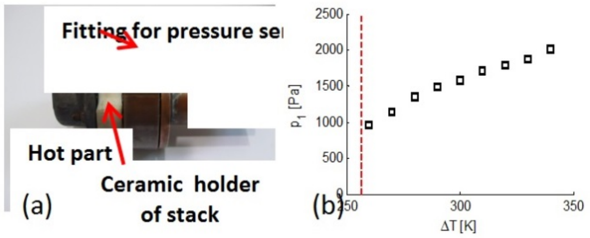

Figure 5. (a) A photograph of 67-mm-long standing-wave engine. (b) Acoustic pressure amplitude recorded as a function of the temperature difference across the stack. The dashed line corresponds to the sound onset (critical ΔT).

5 Concluding Remarks

Heat-sound interactions in acoustic resonators with heat release can result in excitation of acoustic modes. Amplitudes of these oscillations may grow up to the level detrimental or even dangerous for systems such as rocket motors or industrial burners. On the other hand, thermoacoustic motions can be harnessed to generate useful mechanical/electrical power from heat or to produce refrigeration effect. The fundamental physics of heat-sound interactions at low amplitudes is reasonably well understood, but many high-amplitude phenomena still represent challenges for practical control of thermoacoustic instabilities in combustors or for efficient heat-sound energy transformations in thermoacoustic engines. Hence, this area is attractive for future scientific investigations and technological developments. A variety of constructed prototypes of thermoacoustic devices demonstrate a potential promise for widespread application of thermoacoustic technology.

References

[1] Rayleigh, J.W.S., 1945, The Theory of Sound, Dover Publications, New York (re-issue).
[2] Culick F.E.C., 1987, A note on Rayleigh's criterion, Combustion Science and Technology, 56, pp. 159-166.
[3] Raun, R.L., Beckstead, M.W., Finlinson, J.C., and Brooks, K.P., 1993, A review of Rijke tubes, Rijke burners and related devices, Progress in Energy and Combustion Science, 19, pp. 313-364.
[4] Logan, P., Lee, J.W., Lee, L.M., Karagozian, A.R., and Smith, O.I., 1991, Acoustics of a low speed dump combustor, Combustion and Flame, 84, 93-109.
[5] Garrett, S.L. and Backhaus, S., 2000, The power of sound, American Scientist, 88(6), 516-525.
[6] Wang, L.V., 2008, Prospects of photoacoustic tomography, Medical Physics, 35, 5758.
[7] Daschewski, M., Boehm, R., Prager, J., Kreutzbruck, M., and Harrer, A., 2013, Physics of thermo-acoustic sound generation, Journal of applied Physics, 114, 114903.
[8] Matveev, K.I. and Hernandez, R., 2013, Early onset of sound in Rijke tube with abrupt contraction, 21st International Congress in Acoustics, Montreal, Canada, Proceedings of Meetings on Acoustics, **19**, 045088.
[9] Ward, W.C. and Swift, G.W., 1994, Design environment for low-amplitude thermoacoustic engines, Journal of the Acoustical Society of America, 95, pp. 3671-3672.
[10] Harrje, D.T. and Reardon, F.H., 1972, Liquid propellant rocket combustion instability, NASA SP-194.
[11] Flandro, G.A., 1986, Vortex driving mechanism in oscillatory rocket flows, Journal of Propulsion, 2, pp. 206-214.
[12] Natanzon, M.S., 1986, Combustion Instability, Mashinostroenie, Moscow.
[13] Zinn, B. T. and Neumeier, Y., 1997, An overview of active control of combustion instabilities, 35th Aerospace Sciences Meeting and Exhibit, Reno, NV, AIAA paper No. 1997-461.
[14] Candel, S., 2002, Combustion dynamics and control: Progress and challenges, Proceedings of the Combustion Institute, 29, 1-28.
[15] Culick, F.E.C., 2006, Unsteady Motions in Combustion Chambers for Propulsion Systems, RTO AGARDograph AG-AVT-039.
[16] Eisinger, F.L. and Sullivan, R.E., 2007, Prediction of thermoacoustic vibration of burner/furnace systems in utility boilers, ASME PVP Conference, San Antonio, TX, ASME Paper No. 2007-26090.
[17] Angelberger, C., Veynante, D., and Egolfopoulos, F., 2000, LES of chemical and acoustic forcing a premixed dump combustor, Flow, Turbulence and Combustion, 65, 205-222.

[18] Srinivasan, S., Ranjan, R., and Menon, S., 2015, Flame dynamics during combustion instability in a high-pressure, shear coaxial injector chamber, Flow, Turbulence and Combustion, 94(1), 237-262.

[19] Pun, W., 2001, Measurements of Thermo-Acoustic Coupling, PhD Thesis, California Institute of Technology, Pasadena, CA, USA.

[20] Kedia, K.S., Altay, H.M., and Ghoniem, A.F., 2011, Proceedings of the Combustion Institute, 33, 1113-1120.

[21] Kedia, K.S., Altay, H.M., and Ghoniem, A.F., 2011, Proceedings of the Combustion Institute, 33, 1113-1120.

[22] Culick, F.E.C., 1976, Nonlinear behavior of acoustic waves in combustion chambers, Acta Astronautica, 3, 714-757.

[23] Merk, H.J., 1957, Analysis of heat-driven oscillations of gas flows, Applied Scientific Research, A 6, 402-420.

[24] Crocco, L. and Cheng, S.-I., 1956, Theory of Combustion Instability in Liquid Propellant Rocket

[25] Jung, S. and Matveev, K.I., 2010, Study of small-scale standing-wave thermoacoustic engine, Journal of Mechanical Engineering Science, 224(1), 133-141.

[26] Juniper, M.P., 2011, Triggering in the horizontal Rijke tube: non-normality, transient growth and bypass, transition, Journal of Fluid Mechanics, 667, 272-308.

[27] Kulkarni, R., Balasubramanian, K., and Sujith, R.I., 2011, Non-normality and its consequences in active control of thermoacoustic instabilities, Journal of Fluid Mechanics, 670, 130-149.

[28] Rott, N., 1980, Thermoacoustics, Advances in Applied Mechanics, 20, 135-175.

[29] Ward, W.C. and Swift, G.W., 1994, Design environment for low-amplitude thermoacoustic engines, Journal of the Acoustical Society of America, 95, 3671-3672.

[30] Swift, G.W., 2002, Thermoacoustics: A Unifying Perspective for Some Engines and Refrigerators, Acoustical Society of America, Sewickley, PA.

[31] Backhaus, S.N. and Swift, G.W., 2000, A thermoacoustic-Stirling heat engine: Detailed study, Journal of the Acoustical Society of America, 107, 3148-3166.

[32] Gardner, D.L. and Swift, G.W., 2003, A cascade thermoacoustic engine, Journal of the Acoustical Society of America, 114, 1905-1919.

[33] Tijani, M.E.H. and Spoelstra, S., 2011, A high performance thermoacoustic engine, Journal of Applied Physics, 110, 093519.

[34] Swift, G.W., 2002, Thermoacoustics: A Unifying Perspective for Some Engines and Refrigerators, Acoustical Society of America, Sewickley, PA.

[35] Wollan, J.J., Swift, G.W., Backhaus, S.N., and Gardner, D.L., 2002, Development of a thermoacoustic natural gas liquefier, AIChE Meeting, New Orleans, LA.

[36] Berson, A., Michard, M., and Blanc-Benon, P., 2008, Measurement of acoustic velocity in the stack of a thermoacoustic refrigerator using particle image velocimetry, Heat and Mass Transfer, 44(8), 1015-1023.

[37] Matveev, K.I., 2010, Unsteady model for standing-wave thermoacoustic engines, Journal of Non-Equilibrium Thermodynamics, 35(2), 85-96.

[38] Matveev, K.I. and Jung, S., 2011, Modeling of thermoacoustic resonators with non-uniform medium and boundary conditions, ASME Journal of Vibration and Acoustics, 133(3), 031012.

[39] Scalo, C., Lele, S.K., and Hesselnik, L., 2015, Linear and nonlinear modeling of a theoretical travelling-wave thermoacoustic heat engine, Journal of Fluid Mechanics, 766, 368-404.

[40] Wollan, J.J., Swift, G.W., Backhaus, S.N., and Gardner, D.L., 2002, Development of a thermoacoustic natural gas liquefier, AIChE Meeting, New Orleans, LA.

[41] Yu, Z., Backhaus, S., and Jaworski, A.J., 2009, Design and testing of a travelling-wave looped-tube engine for low-cost electricity generators in remote and rural areas, 7th International Energy Conversion Engineering Conference, Denver, CO, AIAA Paper No. 2009-4540.

[42] Gardner, D.L. and Howard, C.Q., 2009, Waste-heat-driven thermoacoustic engine and refrigerator, AAS Conference, Adelaide, Australia.

[43] Symko, O. G., Abdel-Rahman, E., Kwon, Y. S., Emmi, M., and Behunin, R., 2004, Design and development of high frequency thermoacoustic engines for thermal management in microelectronics. Microelectronics Journal, 35, 185-191.